

Visual theorem proving with the Incredible Proof Machine

Interactive Theorem Proving, Nancy
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The idea

Theorem Proving

Theorem Proving

without

Syntax

Theorem Proving

without

Syntax
Linearity

Theorem Proving

without

Syntax

Linearity

Frustration

What does not work?

What does not work?

Pen & Paper

What does not work?

Pen & Paper

Isabelle

What does not work?

Pen & Paper

Isabelle

Coq

A new proof presentation

A new proof presentation

Propositions on Conveyer Belts
Proofs as Assembly Lines

A new proof presentation

Propositions on Conveyer Belts

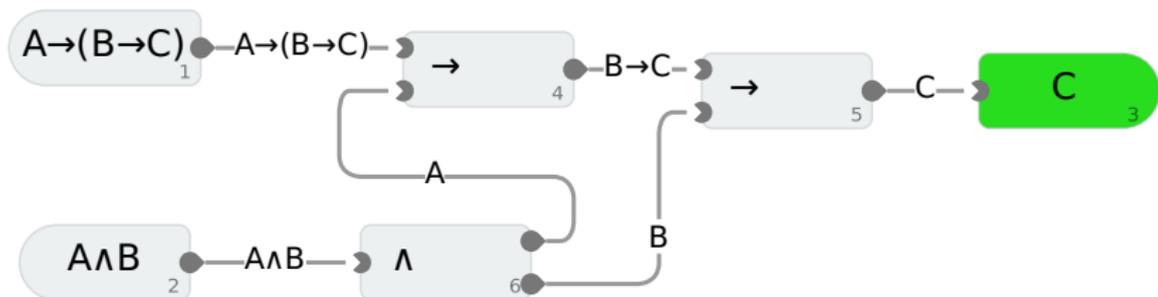
Proofs as Assembly Lines

Proof rules as Machines

producing conclusions from assumptions

The Incredible Proof Machine

A simple proof (uncurrying implication)



Local hypotheses

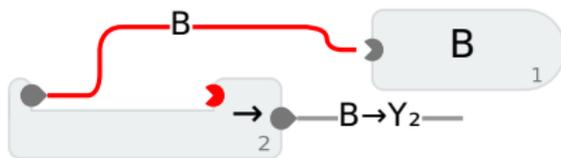
Local hypotheses



Local hypotheses



Local hypotheses

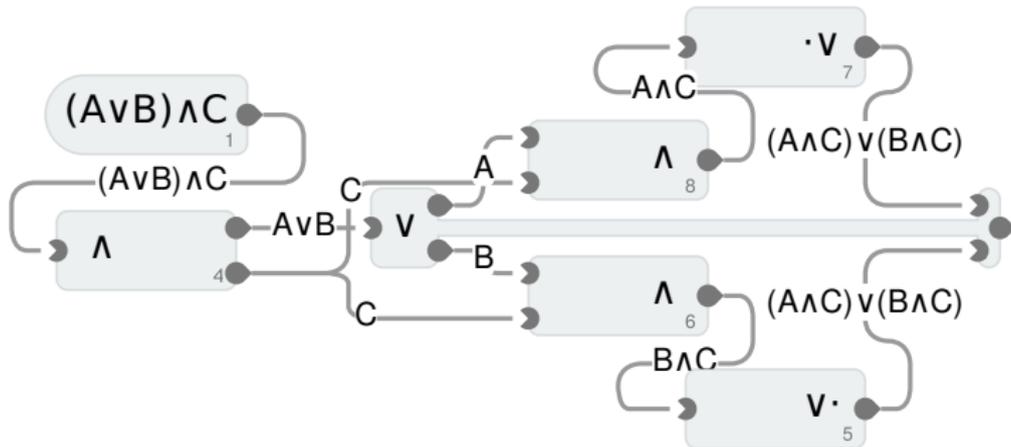


Scopes

Scopes

- Every input port defines a *scope*:

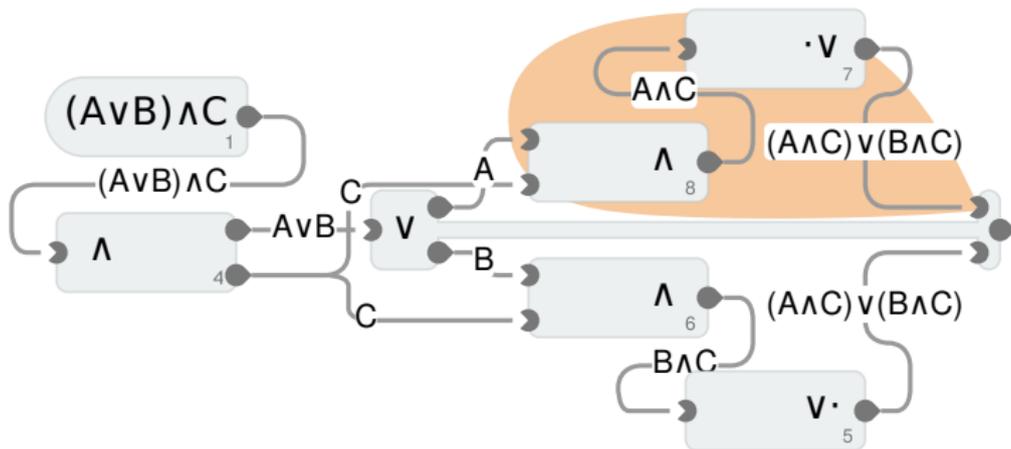
A block is in the scope of an input port iff it is post-dominated by that port.



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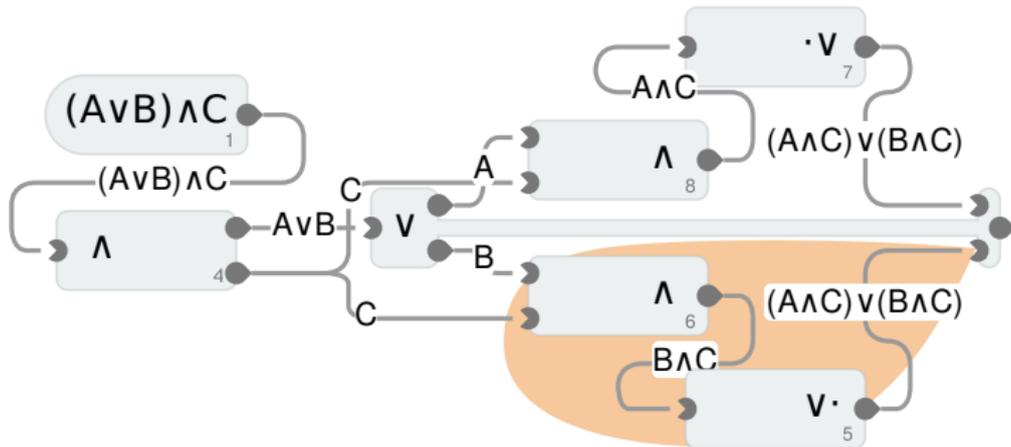
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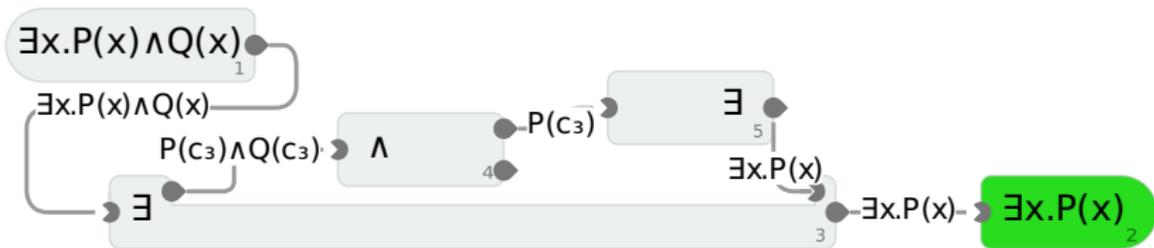
- Every input port defines a *scope*:
A block is in the scope of an input port iff it is post-dominated by that port.
- Each local hypothesis is assigned an input port, and may only be connected to that or a block in that port's scope.

Valid proof graphs are

- Saturated
all input ports are connected
- Acyclic
disregarding local hypotheses
- Well-scoped
all local hypotheses used correctly
- and have a solution
free variables instantiated so that the propositions at the end
of a connection unify

Predicate logic

A proof with \exists



Freshness side conditions



corresponds to

$$\frac{\Gamma \vdash \exists x.P(x) \quad \Gamma, P(c) \vdash Q \quad c \text{ fresh in } \Gamma, P, Q}{\Gamma \vdash Q}$$

Local constants

- Are assigned an input port.
- Are uniquely renamed per instance of a block.
- Must not occur in the instantiation of a block outside the scope of the assigned input port.

Possibly asked questions

Show us your rules? What are your axioms?

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Whatever you want...

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Whatever you want...

The Incredible Proof Machine is a meta logic
and configurable with simple YAML files.

You can do propositional logic, predicate logic,
Hilbert style proofs, STLC typing derivations.

And that is sound? Complete?

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Yes it is.

And that is sound? Complete?

Yes it is.

We modeled such proof graphs in Isabelle
and proved it to be equivalent to natural deduction

Joachim Breitner, Denis Lohner:

The meta theory of the Incredible Proof Machine

The Archive of Formal Proofs, Issue May, 2016,

http://isa-afp.org/entries/Incredible_Proof_Machine.shtml

Can I introduce and use lemmas?
Add definitions?

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Custom blocks
(blocks that encapsulate partial proofs)
serve as lemmas.

Term-level definitions are not yet supported.

Take my money, I want it!

Take my money, I want it!

Keep your money and just go to
<http://incredible.pm/>

The Incredible Proof Machine
is Free Software and runs completely in the browser.
So if you want to use it for your course,
it is easy to modify and host!

Conclusion

The design space of
non-linear
non-textual
interactive
interfaces to theorem proving
is still largely unexplored.

Thank you for your attention.