Mechanical Verification of a Constructive Proof for FLP

according to Hagen Völzer

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Models and Theory of Distributed Systems

Consensus – Motivation

Example

- distributed database
- each at different state
- decide whether to apply transaction

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- exchange messages
- all have to arrive at same decision

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Problem

processes may crash

The FLP Theorem

Theorem (Fischer, Lynch, Paterson, 1985)

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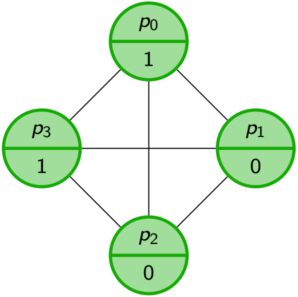
Our Work

- based on the more constructive paper of Völzer
- formalizing this proof in Isabelle/HOL
- ... including "fairness", which was just stated

Consensus

Model

- finite set of sequential processes
- *asynchronous* communication channels between all pairs



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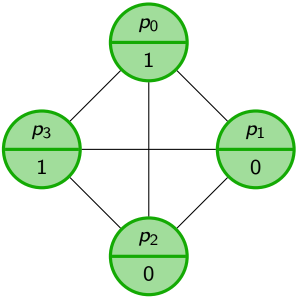

Definition: Binary Consensus

Each process gets an input value from $\{0, 1\}$ and may irrevocably decide on a final output value such that:

- *Agreement*: No two processes decide differently.
- *Validity*: The output value is the input value of some process.
- *Termination*: Each process eventually decides or crashes.

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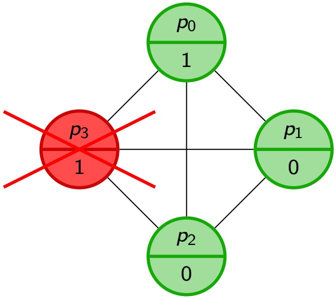

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Definition: Fair Execution

Each message is processed (as long as receiver not crashed).

- unfair execution practically irrelevant

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Theorem (Völzer, 2004)

There is no consensus algorithm such that

- *a process may crash*
- *validity*
- *agreement*
- *every fair execution terminates*

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fundamental result in distributed computing

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Idea of proof

- find invariant that ensures non-decided
- find proper way to extend finite execution, keeping the invariant
- infinite fair run

Initial Lemma

Non-uniform

There are processes p, q such that

- crash of p allows decision 0
- crash of q allows decision 1

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Small error in Völzer's proof

- used same symbol for different configurations
- required adaption in proof

Extension Lemma

Extension Lemma - Völzer's version

For each non-uniform configuration c and each process p there is a configuration c' such that $c \Rightarrow^* c'$ and crash of p in c' allows for both decisions.

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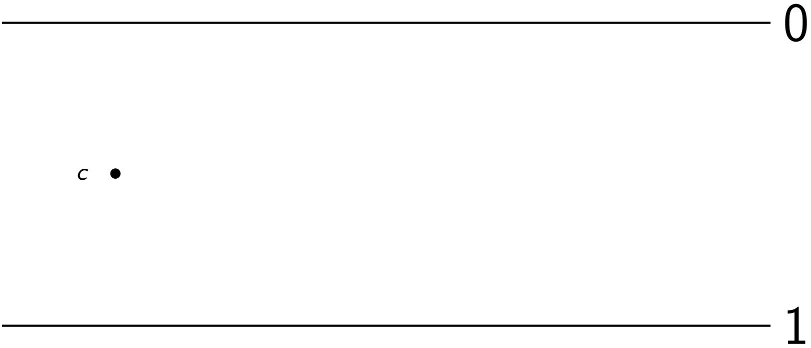
For each non-uniform configuration c and each process p there is a configuration c' such that $c \Rightarrow^* c'$ and crash of p in c' allows for both decisions.

Extension Lemma – our version

- choose message (p, m) – receiver p, content m
- apply Extension Lemma for this p
- can safely consume message (keeping invariant)

all put into single extension

Extension – Picture



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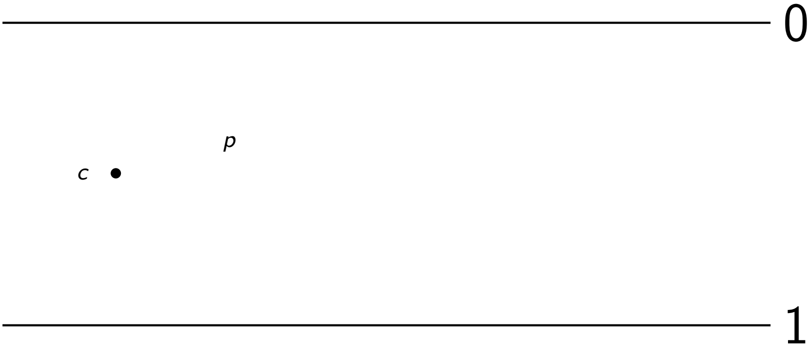

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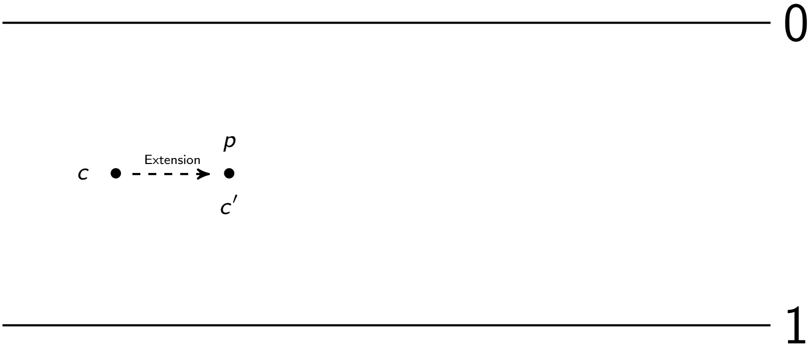

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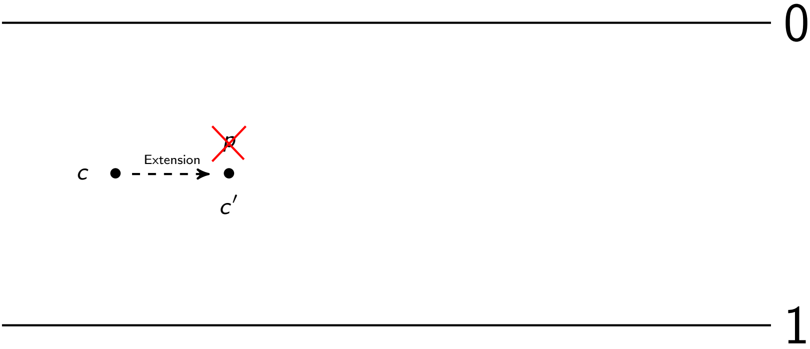

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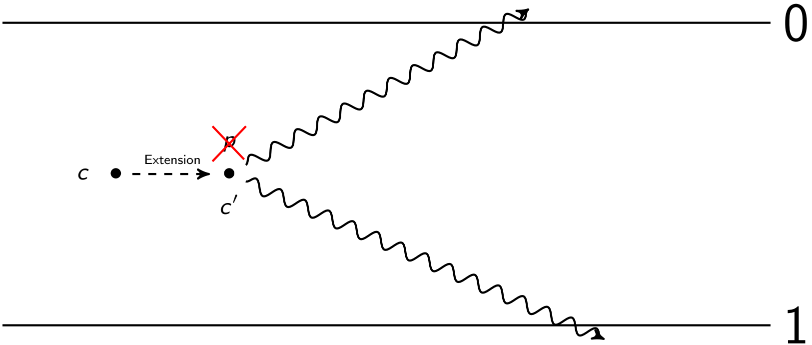

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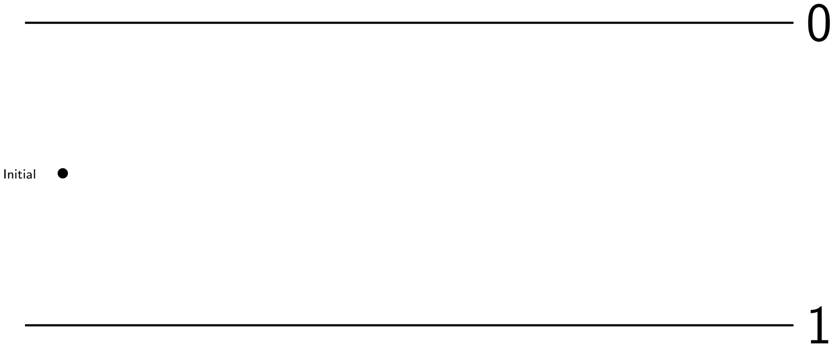
Proof by Völzer

- start with non-uniform initial configuration
- take message with minimal enabling time
- extend execution using Extension Lemma, ending with non-uniform configuration
- repeat this process

Proof Idea



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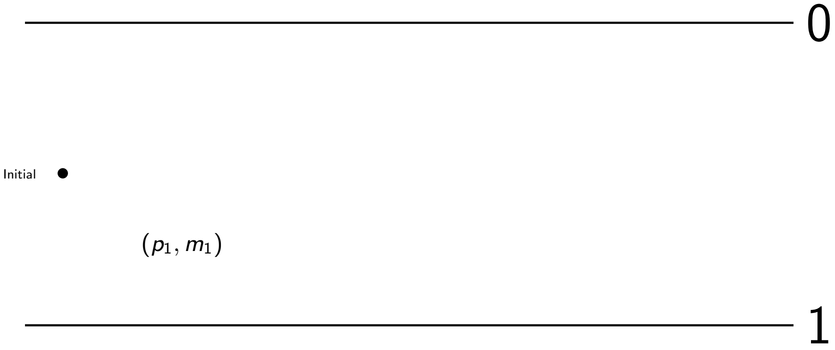

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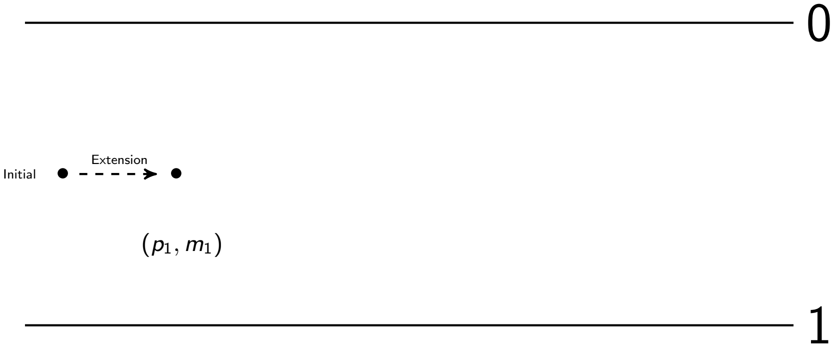

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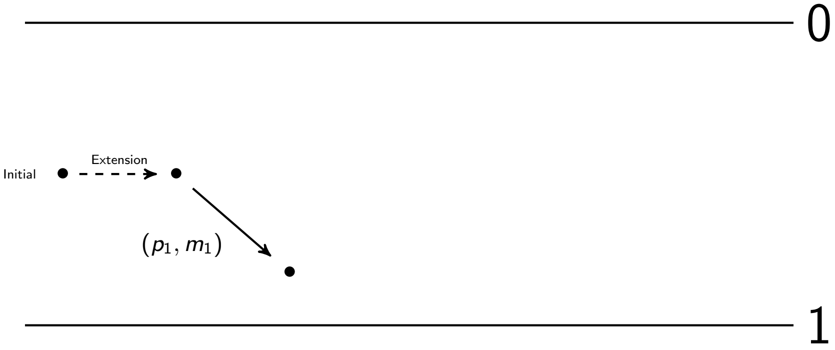

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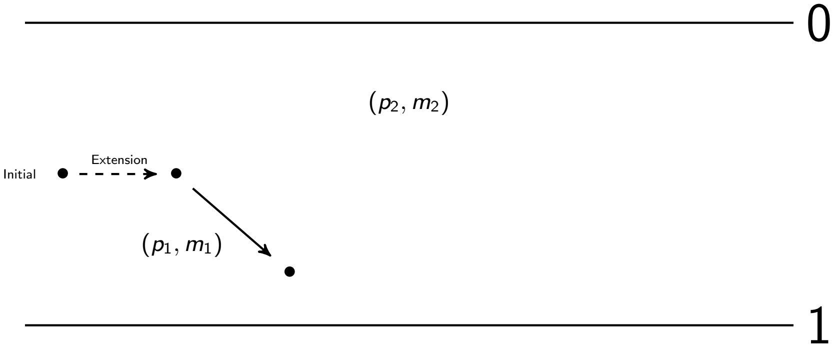

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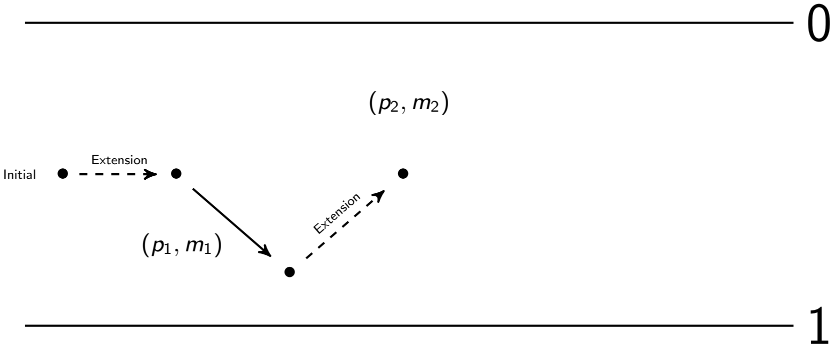

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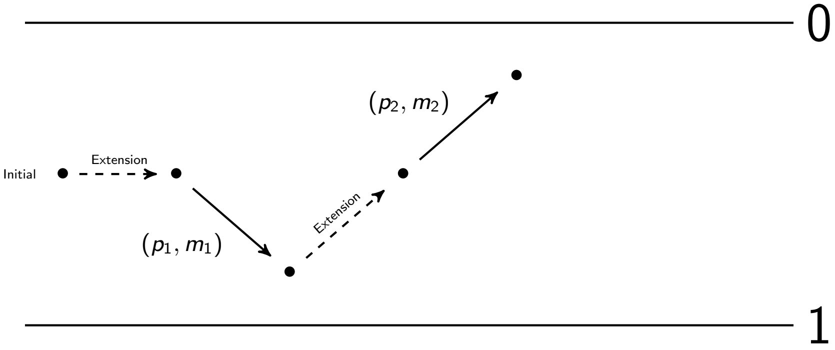

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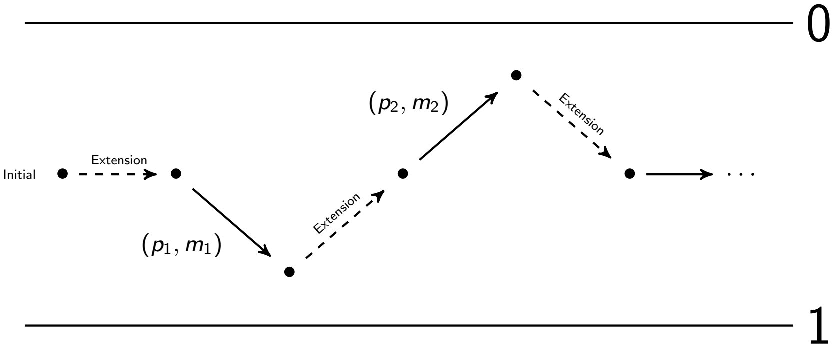

Infinite Executions

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- fairness/correctness defined for single (infinite) execution
- construction yields sequence of finite executions

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```
definition infiniteExecution ::
  "(nat ⇒ (('p, 'v, 's) configuration list))
  ⇒ (nat ⇒ (('p, 'v) message list)) ⇒ bool"
where
  "infiniteExecution fe ft ≡
    ∀ n . execution trans sends start (fe n) (ft n) ∧
      prefixList (fe n) (fe (n+1)) ∧
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Proof of Fairness

Völzer: "We obtain a fair execution
where all processes are correct
and that is always eventually
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Proof of Fairness

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assumes
  Cfg: "initial cfg" "nonUniform cfg"
shows "∃ fe ft.
  (fe 0) = [cfg]
  ∧ fairInfiniteExecution fe ft
  ∧ (∀ n . nonUniform (last (fe n))
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      ∧ (execution trans sends start (fe n) (ft n)))"
proof -
  have BC:  [10 lines]
  obtain fStepCfg fStepMsg where FStep: "∀ cfgList msgList . ∃cfgList' msgList' . [27 lines]
  def Fe: fe == "infiniteExecutionCfg cfg fStepCfg fStepMsg" and [1 lines]

  have BasicProperties: "(∀n. nonUniform (last (fe n))  [26 lines]
  have Fair: "fairInfiniteExecution fe ft" [737 lines]
  show ?thesis proof (rule exI[of _ fe], rule exI[of _ ft]) [6 lines]
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- $2\frac{1}{2}$ pages $\rightarrow$ 4000 LOC
- precise list of preconditions for individual proofs
- proof of fairness
- correctness up to correctness of Isabelle/HOL

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